ANALYSIS OF ENERGY LIMITED SLOTTED COMMUNICATION USING GAME THEORY

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ABSTRACT

In this paper we analyze a system with a number of users communicate using unlicensed spectrum. The time is divided into frames, which are further divided into slots. In each frame a user has limited energy which is allocated to the slots in the frame. Each user tries to maximize his throughput by selecting the best power allocation. The system is modeled using game theory. First an analysis is carried out for a single frame, i.e. a single stage game, and we find the equilibrium points.

1 INTRODUCTION

In recent years we have seen an increase in the use of unlicensed bands. The tremendous success of IEEE 802.11b based devices may be the most striking example. However in unlicensed bands there is no central control of the users. As long as the users adhere to a limited set of rules known as etiquette rules a user can do anything he wishes. The etiquette rules may specify things like maximum transmission duration or maximum output power. These rules should enable coexistence among the users of the spectrum [1].

In licensed spectrum there is usually only one license holder and the owner of the license usually want to maximize the usefulness of the spectrum. This usually translates into maximizing the number of users that can be served or to maximize the total throughput that can be given to the users. The problem for the license holder is to find out how to achieve this and there are numerous radio resource management algorithms that aim at maximizing the spectrum usage. However the key point is that there is only one single objective to be met in licensed spectrum.

For unlicensed spectrum the situation is slightly different. Each user can be assumed to be act in his own interest which may not necessarily be what is best for the total good of all users. The issue is to determine how a user should behave to maximize his utility. Another issue is what the total spectrum usage efficiency will be if all users act selfishly.

There are also related issues that are of concern to the regulator. The previously mentioned issues assume that all users actually follow the etiquette rules. But changing the rules may result in better or worse spectrum usage. The regulator is interested in ensuring as efficient spectrum usage as possible and setting the “correct” rule is of prime interest to the regulator. Another interesting (but out of scope) problem is how to ensure that the users actually follow the rules.

The tool of choice to model interactions between actors with partly contradictory interests is game theory. This mathematical tool was initially used for modeling interaction between business actors [2], but has since been applied to various fields e.g. selfish users in an ALOHA system [3] analysis of the CSMA/CA protocol [4] and power control problems [5].

Finding the rules that best achieve the objective of the regulator is a complicated issue. In this paper we take a bottom up approach and study one example of rules and determine what kind of user behavior they result in.

2 TIMESLOT GAME

Here we model a slotted communication system where the timeslots have duration T. We assume that there are N transmitter receiver pairs and we let G_{ij} denote the propagation loss between transmitter I and receiver j. We let K slots be one single stage of the game. Each user selects transmission power in each of the K timeslots. User i transmits with power P_{ij} in timeslot j. We call this power allocation the strategy of the user. Included in the notion of a single stage game is that a user selects his strategy before the game and it cannot be changed during the game. I.e. there is no possibility for a user to change his actions depending on the actions of the other users.

All users are subject to an energy constraint in the game. Since the duration of all timeslots are equal we can formulate this as the sum of power in all timeslots is limited. Thus:

$$\sum_{j=1}^{K} P_{ij} \leq P_M \tag{1}$$

We should note that in a single stage game there is no point in conserving power and thus in single stage games the users use all available power.

We assume that the users are interested in maximizing the amount of data they can transmit during the game. We assume that the rate that user I achieves in slot k depends on the signal to interference Γ_{ij} of the user in that timeslot according to the following expression:

$$R_{ik} = C\log(1+\Gamma_{ik}) \tag{2}$$

The constant C is a system dependent parameter less than 1. The signal to interference ratio for user i in timeslot k can be calculated using (the well known expression):

$$\Gamma_{ik} = \frac{P_{ik} G_{ii}}{\sum_{j \neq i} P_{jk} G_{ji} + \eta_i} \quad (3)$$

Since each timeslot has the same duration we get the objective function of user i:

$$\max \sum_{i=1}^{K} R_{ik} \quad (4)$$

This problem is similar to allocating power over K channels. The solution to both problems can be found using optimization theory and Lagrange multipliers[6]. The solution is well known in information theory and is commonly known as water filling.

3 SINGLE STAGE GAME

To analyze the game we first determine the Nash equilibrium points. An Nash equilibrium has the characteristic that no user can unilaterally improve his own throughput (utility) by changing strategy [7].

3.1 Two user – two timeslots

We start with the case with two users and timeslots. For user 1 the utility, which should be maximized, becomes:

$$U_1 = C \log\left(1 + \frac{P_{11} G_{11}}{P_{21} G_{21} + \eta_1}\right) + C \log\left(1 + \frac{P_{12} G_{11}}{P_{22} G_{21} + \eta_1}\right) \quad (5)$$

We use the relation that $P_{i2}=P_M-P_{i1}$ and partition the problem into two cases where depending on which timeslot the interference is largest. We first look at the case where the interference is lowest in the first timeslot, i.e. when $P_{21}<P_{22}$. Using waterfilling we find the best response function for user 1:

$$P_{11} = \begin{cases} P_M & if \quad P_{21} \leq \frac{P_M}{2}\left(\frac{G_{21}-G_{11}}{G_{21}}\right) \\ P_M \frac{G_{11}+G_{21}}{2G_{11}} - P_{21}\frac{G_{21}}{G_{11}} & if \quad P_{21} \geq \frac{P_M}{2}\left(\frac{G_{21}-G_{11}}{G_{21}}\right) \end{cases} \quad (6)$$

Note that this is only valid when $P_{21}<0.5P_M$. However the other case is easily analyzed using the same method. We plot the best response for user 1 as a function of the actions of user 2 and obtain the plot in figure 1.

To find the Nash equilibria graphically we can plot the best response function of user 2 in the same graph. In the intersections both users respond to the other user with the best response and thus the point is a Nash equilibrium.

We note that for all values of the pathgain there is a Nash equilibrium when both users allocate half of the power to each timeslot. In addition if $G_{11}<G_{21}$ and $G_{22}<G_{12}$ there is also a Nash equilibrium when both users select one timeslot each to transmit in. Finally there is a interesting case when $G_{11}=G_{21}$ and $G_{22}=G_{12}$. The best response function for user 1 and user 2 becomes $P_{11}=P_M-P_{21}$. In this case we get an infinite number of equilibrium points along the line. Although this case may seem unlikely this case occurs in a cellular environment in the downlink. Another interesting case is when $G_{11}=G_{12}$ and $G_{22}=G_{21}$. This also gives an infinite number of equilibrium points along the "diagonal" of the best response. This situation is what occurs in the uplink of a cell. Therefore for a single cell there are many Nash equilibriums.

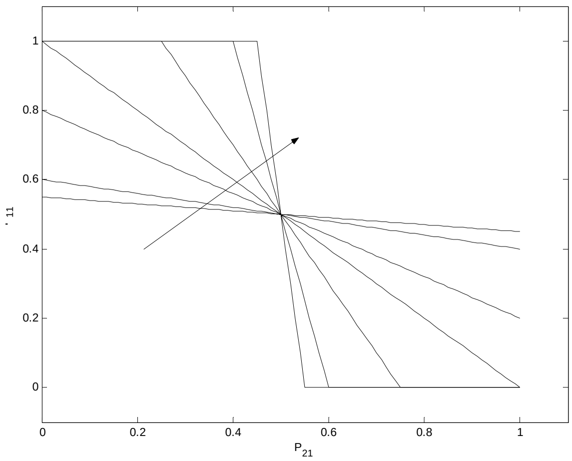

Figure 1. Best response for user 1 as a function of the actions of user 2. The plots are for: G21/G11 = 0.1, 0.2, 0.6, 1, 2, 5 and 10.

To get an idea of the utility for the users in the various equilibrium points we plot the utility in the downlink for both users as a function of P_{11} in figure 2. Note that in each of the points in the plot, user 2 has selected the best response to the action of user 1. Thus for a given P_{11} neither user 1 nor user 2 can do better. The uplink has the same shape but slightly different values.

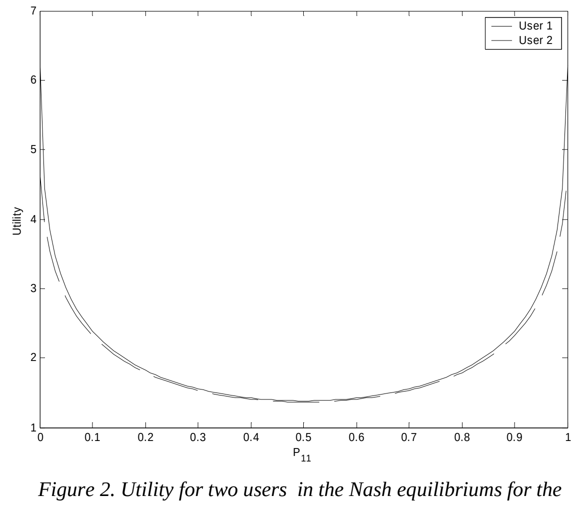

Figure 2. Utility for two users in the Nash equilibriums for the downlink in a single cell.

3.2 Two users – three timeslots

When we increase the number of timeslots to three the problems becomes more difficult to solve. Each user must now select the power in two timeslots since the power in the third timeslot is given by the power limit and the power allocated for the first two timeslots. In total we have a problem in four variables to handle.

We can find the Nash equilibrium points by using the previously used method. First we partition the problem into six partitions and solve for each partition. In one of the partitions we have the following relationship for the ordering of the transmission powers of user 2: $P_{21}<P_{22}<P_{23}$.

Due to the non-linear properties of the waterfilling we get this (slightly complicated) best response for user 1:

In the region:

$$P_{22} \geq P_M \frac{G_{11}}{G_{21}} + P_{21} \tag{7}$$

User 1 allocates power to only 1 timeslot, i.e.

$$P_{11}=P_M,\ P_{12}=P_{13}=0 \tag{8}$$

If (7) is not fulfilled and the following condition is filled:

$$P_{22} < \frac{P_M}{3}\left(\frac{2G_{21}-G_{11}}{G_{21}}\right) - P_{21} \tag{9}$$

User 1 will allocate power to 2 timeslots according to this expression:

$$\begin{aligned} P_{11} &= \frac{P_M}{2} - \frac{P_{21}}{2}\frac{G_{21}}{G_{11}} + \frac{P_{22}}{2}\frac{G_{21}}{G_{11}} \\ P_{12} &= \frac{P_M}{2} + \frac{P_{21}}{2}\frac{G_{21}}{G_{11}} - \frac{P_{22}}{2}\frac{G_{21}}{G_{11}} \end{aligned} \tag{10}$$

Finally if neither (7) nor (9) is fulfilled user 1 will allocate power to all three timeslots according to:

$$\begin{aligned} P_{11} &= \frac{P_M}{3}\left(\frac{G_{11}+G_{21}}{G_{11}}\right) - \frac{G_{21}}{G_{11}}P_{21} \\ P_{12} &= \frac{P_M}{3}\left(\frac{G_{11}+G_{21}}{G_{11}}\right) - \frac{G_{21}}{G_{11}}P_{22} \\ P_{13} &= \frac{P_M}{3}\left(\frac{G_{11}-2G_{21}}{G_{11}}\right) + \frac{G_{21}}{G_{11}}P_{21} + \frac{G_{22}}{G_{11}}P_{21} \end{aligned} \tag{11}$$

The best response function for user 2 in the same partition is easily found essentially by swapping variables. To find the Nash equilibria we essentially have to solve this system of equations:

$$\begin{aligned} (P_{11}, P_{12}, P_{13}) &= f_1(P_{21}, P_{22}, P_{23}) \\ (P_{21}, P_{22}, P_{23}) &= f_2(P_{11}, P_{12}, P_{13}) \end{aligned} \tag{12}$$

Unfortunately the non-linear properties of the equations make solving them difficult. The trick is to assume that the solution has the property that user 1 allocates power to x timeslots and that user 2 allocates power to y timeslots. Then we pick the corresponding equations, solve them and check that the solution is indeed within the limits. There is no equilibrium point where both users allocate power to only one timeslot. The conditions for a Nash equilibrium of the various kinds are outlined in table 1.

Table 1: Sufficient conditions for the existence of various Nash equilibria.

Slots User1	Slots User2	Conditions
1	1	No equilibrium point
1	2	$G_{11}/G_{21}<½$; $G_{12}/G_{22} > ½$
1	3	$G_{11}/G_{21} < G_{12}/G_{22} < ½$
2	1	$G_{11}/G_{21}<2$; $G_{12}/G_{22} > 2$
2	2	$½ < G_{11}/G_{21} < G_{12}/G_{22} < 2$
2	3	$G_{11}/G_{21} < G_{12}/G_{22} < 2$
3	1	$2 < G_{11}/G_{21} < G_{12}/G_{22}$
3	2	$½ < G_{11}/G_{21} < G_{12}/G_{22}$
3	3	Always present

It should be noted that most of the equilibrium points outlined above occur when the signal and interference paths are of the same order. For example in the downlink in a single cell where G11=G21 and G22=G12 we get equilibrium points where both users allocate power to two or three timeslots.

3.3 N users K timeslots

Unfortunately the solution method used in the previous section quickly becomes cumbersome. When more users are involved we have to resort to numerical methods.

To find the equilibrium points we use a Monte-Carlo style simulator. We start with a random power allocation for all users and let each user in update their strategies as a response to the actions of the other users, one user at a time. If the algorithm converges we have found a Nash equilibrium. For each of the cases described below we run 10000 experiments. We consider permutations of the power allocation to be equivalent. For example in the two timeslot case say that user 1 and 2 allocates full power to one timeslot each. We consider this to be the same point regardless if it is user 1 or 2 that transmit in the first slot.

We consider three main cases. The first is a square service area (1x1) where the transmitters are uniformly distributed over area and the receivers are located at a distance of 0.3 from the transmitters in a random direction. The second and third cases are the uplink and downlink of a single circular cell with uniform user distribution. Note that we only use one instance of the environments when searching for equilibrium points. In each of the three main cases we look at: more users than timeslots (10 users 7 timeslots), more timeslots than users (7 users 10 timeslots) and the same number of users and timeslots (10 of each).

The results are outlined in table 2. "Many" denotes that the algorithm essentially found one equilibrium point for each experiment. When studying the scattered pairs scenario the equilibrium points showed little regularity. Some users allocated full power to one slot and others spread the power more evenly over the timeslots. Note that permutations of the timeslot allocation are counted as the same solution, thus the single equilibrium point in the 10 users 7 timeslot case actually correspond to 7!=5040 equilibrium points.

Table 2. Number of Nash equilibrium points found when searching using numerical methods.

	10 users 7 timeslots	10 users 10 timeslots	7 users 10 timeslots
Scattered pairs	1	2	4
Uplink	Many	Many	Many
Downlink	Many	Many	Many

The drawback with this method is that there is no guarantee that we find all equilibrium points. Especially some of the equilibrium points are unstable. Thus a slight deviation of the power of one user will cause the other users to move away from the point. This means that if we do not exactly hit the point in the initial stage the point will not be found by the numerical search. For example the equilibrium point where all users allocate the same power to all slots was not found in any of the experiments.

4 CONCLUSIONS

First of all the results in this paper illustrates the difficulty to analyze this type of problems. For a few users and timeslots it is possible to use analytical tools, but with the addition of only a few more users or timeslots the calculations become quite tedious. This is (unfortunately) in line with other radio resource management problems. It should also be noted that we have not added the usual complication, e.g. fading and other random effects. Also the lack of “nice” numbers indicates that the solutions to the problems will be clean expressions.

For the case with scattered users there seems to be a limited amount of equilibrium points, but there seems to be more as the number of available timeslots increase relative to the number of users.

There seems to be a special case where all users communicate with an access point in a cellular environment. This has to do with the properties of the gain matrix that has identical rows or columns. In this case there seems to be infinitely many equilibrium points. The interesting property here is that it gives a single user the possibility to disturb the communication in the cell for the other users. This gives a good negotiation position in a repeated game and this situation should be further analyzed using a model of repeated games.

Finally it should be noted that assumptions made to make the problem tractable also makes the application of the results difficult. In a practical system there would be no artificial boundaries for a single stage game. Instead users would know the actions of the other users after a few timeslots and could then adapt their actions accordingly.

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REFERENCES

[1] PTSFS 2004 ”Post- och telestyrelsens föreskrifter om undantag från tillståndsplikten för vissa radiosändare” *Post och Telestyrelsen* 2004

[2] M. D. Davis “Game theory: a nontechnical introduction” *Dover Publishing, New York,* 1997

[3] A. B. MacKenzie and S. B. Wicker, “Stability of multipacket slotted Aloha with selfish users and perfect information,” in *Proceedings of IEEE INFOCOM*, 2003.

[4] M. Cagalj, S. Ganeriwal, I. Aad and J.-P. Hubaux “On cheating in CSMA/CD Ad Hoc Networks” *EPFL-IC technical report No. IC/2004/27* http:// icwww.epfl.ch/publications/documents/ IC_TECH_REPORT_200427.pdf (accessed 2004-05-25)

[5] C. U. Saraydar, N. B. Mandayam, and D. J. Goodman, “Efficient power control via pricing in wireless data networks,” *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 291–303, February 2002.

[6] T. M. Cover and J. A. Thomas “Elements of information theory” *Wiley, New York,* 1991

[7] M. J. Osborne and A. Rubinstein “A course in game theory” *MIT Press, Cambridge* 1994